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Solution by the PROPOSER.

The probable error for a , $=r_1/a$; for b , r_1/b ; for c , r_1/c .

\therefore Error of volume in length $=bcr_1/a$.

Error of volume in width $=acr_1/b$.

Error of volume in thickness $=abr_1/c$.

The probable error of volume = square root of the sum of the squares of these three errors.

\therefore Probable error $=\sqrt{[(ab^2c^2 + a^2bc^2 + a^2b^2c)r^2]} = r_1/[abc(ab + ac + bc)]$.

MISCELLANEOUS.

124. Proposed by J. W. YOUNG, Graduate Student, Cornell University, Ithaca, N. Y.

Prove that the general value of θ , which satisfies the equation

$$(\cos\theta + i\sin\theta)(\cos2\theta + i\sin2\theta)\dots \text{to } n \text{ factors} = 1 \text{ is } \frac{4m\pi}{n(n+1)};$$

where m is any integer ($i = \sqrt{-1}$).

Solution by G. W. GREENWOOD, A. M., McKendree College, Lebanon, Ill.; LON C. WALKER, A. M., Leland Stanford Jr. University, Cal. and J. SCHEFFER, A. M., Hagerstown, Md.

$$1 = (\cos\theta + i\sin\theta)(\cos2\theta + i\sin2\theta)\dots(\cos n\theta + i\sin n\theta) = (\cos\theta + i\sin\theta)^{1+2+\dots+n}$$

$$= (\cos\theta + i\sin\theta)^{\frac{1}{2}n(n+1)} = \cos \frac{n(n+1)\theta}{2} + i \sin \frac{n(n+1)\theta}{2}.$$

$$\therefore \frac{n(n+1)\theta}{2} = 2m\pi, \text{ where } m \text{ is any integer; } i. e. \theta = \frac{4m\pi}{n(n+1)}.$$

Also solved by G. B. M. ZERR.

125. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Assume $m = nt + \varepsilon - \omega$, thus giving $v = m + \varepsilon \sin v$ as the relation connecting the mean and eccentric anomalies, then express $x = a \cos v$, $y = b \sin v$, and $r = a(1 - e \cos v)$ by a Fourier series in terms of m .

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

If $y_1 = z + x\varphi(z)$, we get by Lagrange's Theorem,

$$f(y_1) = f(z) + x\varphi(z)f'(z) + \frac{x^2}{1.2} \frac{d}{dz} \{[\varphi(z)]^2 f''(z)\} + \frac{x^3}{1.2.3} \left(\frac{d}{dz}\right)^2 \{[\varphi(z)]^3 f''(z)\} + \text{etc., etc.}$$

From $v = m + \varepsilon \sin v$, $y_1 = v$, $z = m$, $x = e$, $\varphi(y) = \sin v$.

Now $f(v) = v$ and $f'(v) = 1$.

$$\therefore y_1 = z + x \sin z + \frac{x^2}{1.2} \frac{d}{dz} (\sin^2 z) + \frac{x^3}{1.2.3} \left(\frac{d}{dz} \right)^2 (\sin^3 z) + \text{etc.}$$

$$\begin{aligned} \therefore y_1 &= z + x \sin z + \frac{x^2}{1.2} \frac{d}{dz} \left(\frac{1 - \cos 2z}{2} \right) + \frac{x^3}{1.2.3} \left(\frac{d}{dz} \right)^2 \left(\frac{3 \sin z - \sin 3z}{4} \right) \\ &+ \frac{x^4}{1.2.3.4} \left(\frac{d}{dz} \right)^3 \left(\frac{3 - 4 \cos 2z + \cos 4z}{8} \right) + \text{etc.} \\ &= z + x \sin z + \frac{1}{2} x^2 \sin 2z + \frac{1}{8} x^3 (3 \sin 3z - \sin z). \end{aligned}$$

$$\begin{aligned} \therefore v &= m + e \sin m + \frac{1}{2} e^2 \sin 2m + \frac{1}{8} e^3 (3 \sin 3m - \sin m) \\ &+ \frac{1}{8} e^4 (2 \sin 4m - \sin 2m) + \text{etc.} = m + e \sin v. \end{aligned}$$

$$\therefore \sin v = \sin m + \frac{1}{2} e \sin 2m + \frac{1}{8} e^2 (3 \sin 3m - \sin m) + \frac{1}{8} e^3 (2 \sin 4m - \sin 2m) + \text{etc.}$$

To develop $(1 - e \cos v)$ in terms of m : Let $f(y_1) = 1 - e \cos y_1$, $f'(y_1) = e \sin y_1$.

$$\therefore 1 - e \cos y_1 = (1 - e \cos z) + x \sin z (e \sin z) + \frac{x^2}{1.2} \frac{d}{dz} (\sin^2 z \cdot e \sin z) + \text{etc.}$$

Performing the operations as before we get after substituting v for y_1 , m for z , e for x ,

$$\begin{aligned} 1 - e \cos v &= 1 - e \cos m + \frac{1}{2} e^2 (1 - \cos 2m) + \frac{1}{8} e^3 (3 \cos m - 3 \cos 3m) \\ &+ \frac{1}{8} e^4 (\cos 4m - \cos 2m) + \text{etc.} \end{aligned}$$

$$\begin{aligned} \therefore \cos v &= \cos m + \frac{1}{2} e (\cos 2m - 1) + (3e^2/8) (\cos 3m - \cos m) \\ &+ \frac{1}{8} e^3 (\cos 2m - \cos 4m) + \text{etc.} \end{aligned}$$

$$\begin{aligned} \therefore \sin v &= A \sin m + B \sin 2m + C \sin 3m + D \sin 4m + \dots \\ \cos v &= -\frac{1}{2} e + A_1 \cos m + B_1 \cos 2m + C_1 \cos 3m + D_1 \cos 4m + \dots \end{aligned}$$

where $A, B, C, D, \dots, A_1, B_1, C_1, D_1, \dots$ are each a series in powers of e .

$$\begin{aligned} \therefore x = a \cos v &= -\frac{1}{2} a e + a A_1 \cos m + a B_1 \cos 2m + a C_1 \cos 3m + a D_1 \cos 4m + \dots \\ y = b \sin v &= b A \sin m + b B \sin 2m + b C \sin 3m + b D \sin 4m + \dots \\ r = a(1 - e \cos v) &= a \left[1 + \frac{1}{2} e^2 - e A_1 \cos m - e B_1 \cos 2m - e C_1 \cos 3m \right. \\ &\quad \left. - e D_1 \cos 4m + \dots \right]. \end{aligned}$$

126. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

The declination of a certain fixed star is $12^\circ 40'$. Its altitude was observed one day to be $16^\circ 40'$. Three hours and twenty-four minutes later it was found to be $40^\circ 20'$. Find the latitude of the place of observation.